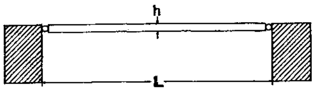

Fig. 1 Plate strip

The problem now will be considered in more detail. In Ref. 2, for instance, it is stated (in connection with a circular plate problem) that "For thin plates the temperature may be approximated by $T_0(r,\theta,t) + zT_1(r,\theta,t)$ where the effects of the two terms may be superimposed if the deflections are small and the material is linearly elastic." Newman and Forray place no restriction on the magnitude of T_0 or the corresponding forces caused by this term. The assumption of inequality (2) is needed, however. An example now will be given in order to indicate how restrictive this inequality can be. Consider a "two-dimensional" plate strip for simplicity (Fig. 1). The plate is pinned between two immovable supports and is of length L, thickness h, and made of aluminum. It is loaded by a uniform cooling of $T_0°$ and by a system of thermal moments. For a plate 28 in. long and 0.100 in. thick which deflects in an approximate half-sine wave, it is found that the superposition principle will give an answer in error by 100% if T_0 is as large as 1°F. In other words, neglecting the midplane restoring forces due to a 1° temperature drop will cause the calculated deflection to be twice as large as the correct value. For an accurate answer, T_0 would have to be limited to much less than 1°F in this case! (It is realized that a temperature rise of only 1°F would cause this same plate to buckle. This is a different question, however, and is not of interest here.)

Of course, counter-examples can be constructed which would be less restrictive, particularly for thicker plates and for plates free to expand. For thin plates in general, however, it appears that the superposition is applicable only to cases where the midplane stresses are so low that they may as well be neglected in the determination of the maximum stresses. If this is the case, then the superposition is not needed.

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Reply by Authors to William J. Anderson

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Anderson states in his note that the inclusion of membrane restoring forces in the plate equilibrium equation is "generally considered a part of classical linear plate theory." This statement is debatable, since Eq. (1) cannot be derived from variational principles without the inclusion of nonlinear terms in the strain-displacement relations. Novozhilov[1] remarks that when these nonlinear terms are neglected "one obtains the formulas of the classical theory of plates." Furthermore, regardless of what the expression "classical plate theory" means, the present authors clearly stated, in the references cited by Anderson, that superposition of the bending and in-plane problems is being used. It is true that the limitations of this superposition technique have not been established. However, the determination of the range of applicability of this method often presents a very difficult task. In the class of problems investigated, the temperature and hence the membrane forces are permitted to vary over the plate planform. If the deflections are small, Anderson's requirement that the inequality (2) be satisfied *throughout* the plate is a sufficient but not necessary condition for the applicability of superposition. It is far too restrictive, since superposition still may be used when (2) is satisfied on some average basis rather than pointwise. For example, superposition should yield accurate solutions (with interior bending and membrane stresses of the *same* order of magnitude) for a wide variety of nonuniform heating problems in which the edges are unrestrained in the plane of the plate. These are very practical problems, since important structural components usually are designed to permit thermal expansion. As a guide to the reader, superposition generally can be considered valid when the absolute ratio of the generated edge thrust to the buckling thrust is small compared to unity.

No one will deny the inaccuracy of superposition for an axially restrained thin strip. The authors never intended, however, that the technique be used indiscriminately.

In closing, it must be remarked that Refs. 2–8 present solutions to many thermoelastic in-plane and bending problems. Anderson neglects to mention that either type of problem is of importance in itself.

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Comment on "Error Matrix for a Flight on a Circular Orbit"

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In a recent technical note,[1] Wisneski derives the well-known solutions of the linear perturbation differential

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